Contribution to the application of Matlab simulation in mechanical engineering innovation, specifically in modeling mechanical systems with a focus on manipulators

Darina Hroncová[1]

[1] Faculty of Mechanical Engineering, Department of Mechatronics, Technical University of Kosice; Letná 9, 042 00 Košice, Slovakia; darina.hroncova@tuke.sk

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Abstrakt: The presented article is dedicated to utilizing Matlab software to address kinematics and dynamics challenges in a basic model of a two-link robotic arm. The introduction offers an overview of computer programs employed in the realm of kinematics and dynamics within mechanical systems. The application of the Matlab program is demonstrated in solving both inverse and direct kinematics problems, along with the inverse problem of dynamics.

Keywords: simulation, Matlab, kinematics, dynamics, direct, inverse, trajectory

1. INTRODUCTION

Computer modeling is advancing in various sectors of industrial production. It is extensively applied in the automotive industry and beyond, focusing on reducing the development time for new product models and streamlining the development and production processes. In the automotive sector, for instance, it optimizes the acceleration and performance of vehicles on designed virtual paths.

It is commonly utilized in engineering tasks where cost-effective insights into the behavior of modeled systems are required. It allows the alteration of model parameters to observe the dependency of their influences on the final solution. The advantage of computer modeling lies in its speed and great flexibility in addressing various problems [1-5].

In computer modeling, numerous software options are available, including the following programs [6-8]:

Matlab: A programming environment enabling numerical calculations, modeling, and simulations [9].
Maple: A comprehensive computational software allowing analytical and numerical calculations, graphical representation of results, and the creation of documentation describing the work process.
Mathematica: A program focused on numerical and matrix tasks in various engineering domains.
Matlab/Simulink: This module of Matlab is developed for creating and solving dynamic systems using block diagrams.
Matlab/SimMechanics: An extension of Matlab developed for solving the kinematics and dynamics of rigid bodies, compatible with older Matlab versions.
Matlab/Simscape: An extension of Matlab designed as a replacement for the aforementioned SimMechanics, addressing kinematics and dynamics of rigid bodies.
Dynast: Enables simple mathematical calculations and simulations.
MSC Adams: Utilizes an object-oriented programming environment with graphical simulation capabilities. Systems of bodies are defined directly through body geometry, kinematic constraints, force effects, and motion generators. Given its predefined program, MSC Adams proves instrumental in shortening testing durations, mitigating error risks, acquiring essential input data, and providing solutions for highly complex mechanical systems with multiple degrees of freedom.

2. MANIPULATOR MODEL

An essential part of analyzing robot models is the complete kinematic model of the mechanical system, providing all necessary kinematic parameters for both the dynamic model of the system (force application, link loading, sizing) and control requirements. This primarily involves the positional trajectory of individual mechanism links. The position of the links is generally described using so-called generalized coordinates. In robotics, the term joint variables is often used, indicating the rotation or translation of individual motion axes. Figure 1b) illustrates the model of a mobile robot with a two-link manipulator. Figure 1c) displays the model of a stationary robot, where the manipulator arm is mounted on a fixed base. From a kinematic structure perspective, it represents an open kinematic chain (Figure 1a), [1-5].

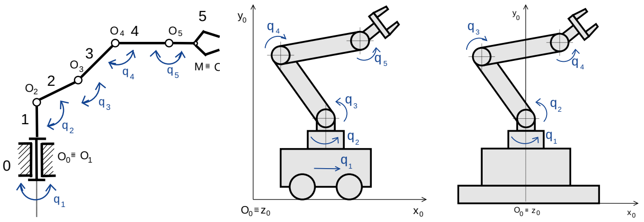
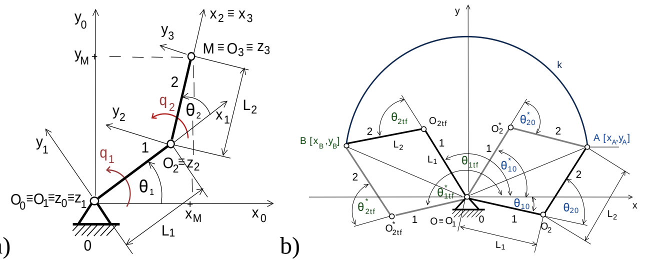


Figure 1. Manipulator model: a) open kinematic chain, b) two-link robotic arm on a mobile chassis, c) two-link robotic arm on a fixed chassis

The structure of an open kinematic chain with five degrees of freedom is shown in Fig.1 a). Individual links are connected by rotational joints. The first link is fixed to a stationary base. The movement of the links is described by generalized coordinates q_i in Fig.1 a), which are q_1 to q_5. Each link is associated with a local coordinate system O_i, x_i, y_i, z_i. Since the movement of the links in the kinematic chain of the manipulator model involves simultaneous motions, we are interested in the motion of the local coordinate systems of each link with respect to the reference coordinate system O_0, x_0, y_0, z_0, connected to the base 0. When solving the forward kinematics according to Fig.2 a), we determine the position of the selected point $M(x_M,y_M)$ with respect to the reference coordinate system O_0, x_0, y_0, z_0 with known angular rotations of the arms θ_1 and θ_2 [1-5].



a) b)

Figure 2. Manipulator model a) two-joint robotic arm, b) angles of rotation θ_{10}, θ_{20}, θ^*_{10} and θ^*_{20} for a known position of the end point $A(x_A,y_A)$

We describe the position of point M (Fig. 2a) by its coordinates x_M and y_M with respect to the coordinate system of the base 0 using equations [1-5]:

$$x_M = L_1 \cdot cos\theta_1 + L_2 \cdot cos\,(\theta_1 + \theta_2) \qquad (1)$$

$$y_M = L_1 \cdot sin\theta_1 + L_2 \cdot sin\,(\theta_1 + \theta_2) \qquad (2)$$

In the field of manipulator kinematics, we encounter the solution of the inverse kinematics problem, where, according to Fig. 2b), with a known position of point $A(x_A,y_A)$, we need to determine the rotation angles of the arms θ_{10}, θ_{20}, θ^*_{10} and θ^*_{20}. The parameters of the points for the solved positions are in Table 1.

Table 1. Coordinates x,y of the points A, B, C, D, E, F, G, H, I, J

	A	B	C	D	E	F	G	H	I	J
x (m)	0.41	-0.19	0.22	0	0.22	-0.22	0.22	-0.22	0.41	-0.41
y (m)	0.0	0.22	-0.19	0.41	-0.19	0.19	-0.19	0.19	0	0

In this case, we determine the angles θ_{10}, θ_{20}, $\theta^*_{10} = \theta_{1tf}$, and $\theta^*_{20} = \theta_{2tf}$ in Table 2 by solving the system of equations (1) and (2) using the Matlab software for the arm positions L_1 and L_2 with the endpoint A according to Fig. 3a):

```
x0=0.41          %xA
y0=0.0           %yA
L1=0.22
L2=0.19
syms theta10 theta20
E10 = L1*cos(theta10)+L2*cos(theta10+theta20)-x0;
E20 = L1*sin(theta10)+L2*sin(theta10+theta20)-y0;
[theta10, theta20] = solve(E10,E20);
theta10 = double(theta10*(180/pi))
theta20 = double(theta20*(180/pi))
```

Table 2. Sizes of angles between points A-B trajectory k_1,C-D (k_2), E-F (k_3), G-H (k_4), I-J (k_5)

	A-B (k_1)	C-D (k_2)	E-F (k_3)	G-H (k_4)	I-J (k_5)
Θ_{10}[°]	0	0	0	0	0
Θ_{20}[°]	0	-90	-90	-90	0
Θ_{1tf}[°]	90	90	180	180	180
Θ_{2tf}[°]	90	0	270	-90	0

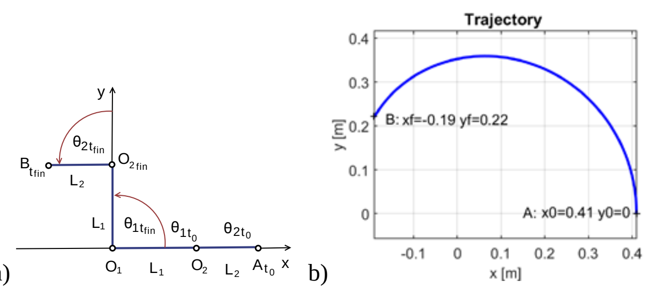
Exploring the movement of the manipulator arm in a position for maximum extension along the x-axis and minimum extension along the x-axis (Fig. 3a) is detailed in the following section.

With the arms of the manipulator positioned as shown in Fig. 3a), we determine the angles of rotation and plot the trajectory k_1 (Fig. 3b). This trajectory is obtained during the motion of the arms from the initial position marked by point A to the final position denoted by point B for the second arm (Table 1). The resulting trajectory k_1 is illustrated in Fig. 3b). Angle magnitudes are provided in Table 2. We determine the trajectory using a 5th-degree polynomial. Coefficients a_6 and b_6 are determined from known initial angles of both arms (Table 3).

Table 3. Sizes of coefficients between points A-B (k_1), C-D (k_2), E-F (k_3), G-H (k_4), I-J (k_5)

	A-B (k_1)	C-D (k_2)	E-F (k_3)	G-H (k_4)	I-J (k_5)
a_1	0.2945	0.2945	0.5890	0.5890	0.5890
a_2	-1.4726	-1.4726	-2.9452	-2.9452	-2.9452
a_3	1.9635	1.9635	3.9270	3.9270	3.9270
a_6	0	0	0	0	0
b_1	0.2945	0.2945	1.1781	0	0
b_2	-1.4726	-1.4726	-5.8905	0	0
b_3	1.9635	1.9635	7.8540	0	0
b_6	0	-1.5708	-1.5708	-1.5708	0

Initial and final angular velocity, as well as angular acceleration, are assumed to be zero. Under these conditions, we find coefficients a_4 , a_5 , b_4 and b_5, which are set to zero. Solving the system of equations yields the remaining coefficients a_1 , a_2 , a_2 , b_1 , b_2 and b_3 (Table 3).



a) b)

Figure 3. a) Arm position at point A and point B, b) trajectory k_1 during the movement from point A to point B

The course of angular parameters during the movement of the arms from the initial position A to the final position B is illustrated in Fig. 4.

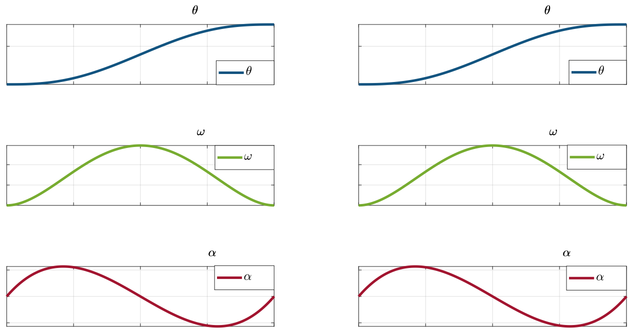
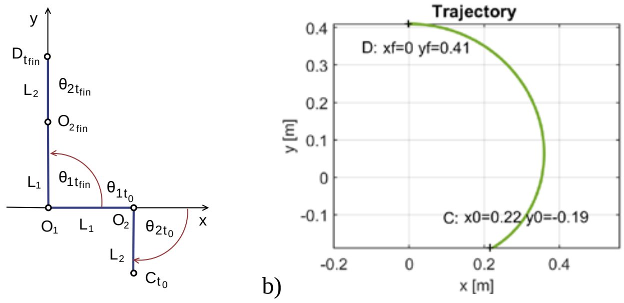


Figure 4. Angular parameters during the motion from the position of the arms defined by point A to the position of the arms defined by point B, as described by the curve k_1

The examination of the manipulator arm's movement at the position for maximum extension in the y-axis and minimum extension in the y-axis (Fig. 5a) is further described. At the arm's position on Fig. 5a), after determining the angular rotations, we define its trajectory k_2 (Fig. 5b). This trajectory is obtained during the motion of the arms from the initial position defined by point C to the final position defined by point D of the second arm (Table 1). The result is the course of trajectory k_2 in Fig. 5b).



a) b)

Figure 5. a) Arm positions at maximum y and minimum y, b) trajectory k_2 during the movement from point C to point D

The progression of angular parameters during the movement of the arms from the initial position C to the final position D is illustrated in Figure 6.

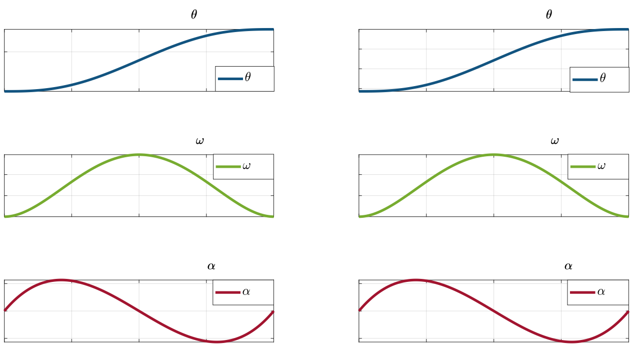
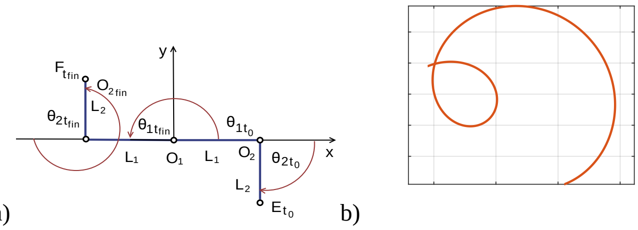


Figure 6. Angular parameters during the movement from the arm position defined by point C to the arm position defined by point D, as described by the curve k_2

The investigation of the manipulator arm motion in the position for minimal arm extension along the -y axis and minimal extension along the +y axis (Fig. 7a) follows. This involves the movement from position E to position F.

When the arms of the manipulator are in the position shown in Fig. 7a), after determining the rotational angles, we determine its trajectory k_3 (Fig. 7b). We obtain this trajectory during the movement of the arms from the initial position specified by point E to the final position specified by point F of the second arm (Table 1). The result is the trajectory k_3 in Fig. 7b).



a) b)

Figure 7. a) Arm positions at minimum -y and maximum +y, b) trajectory k_3 during the motion from point E to point F

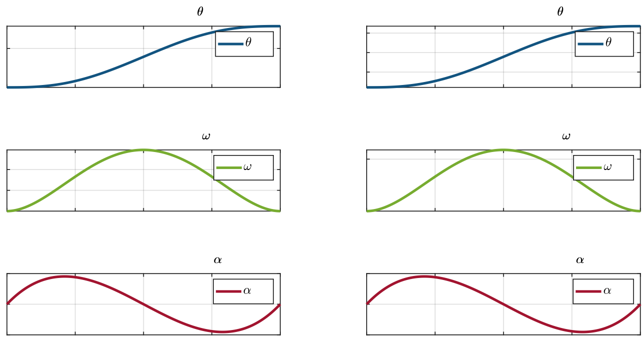
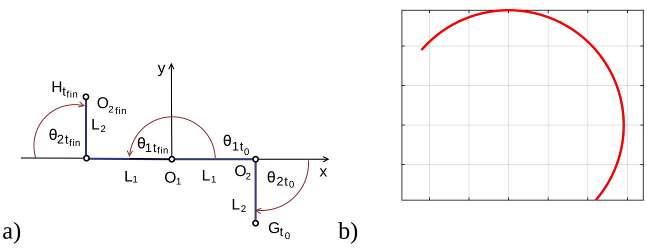


Figure 8. Angular parameters during the motion from the position of the arms defined by point E to the position defined by point F, as described by the curve k_3

Analyzing the movement of the manipulator's arm in the position for minimal extension along the -y axis and minimal extension along the +y axis (Fig. 9a) is described next. This involves motion from position G to position H.

When the manipulator's arms are in the position shown in Fig. 9a), after determining the angular rotations, we can establish its trajectory k_4 (Fig. 9b). This trajectory is obtained as the arms move from the initial position, defined by point G, to the final position, defined by point H of the second arm (Table 1). The resulting depiction of trajectory k_4 is illustrated in Fig. 9b).



a) b)

Figure 9. a) Arm position at minimum -y and +y, b) trajectory k_4 during the movement from point G to point H at angle θ_{2tfin}=-90°

The progression of angular values during the motion of the arms from the initial position G to the final position H is shown in Fig. 10.

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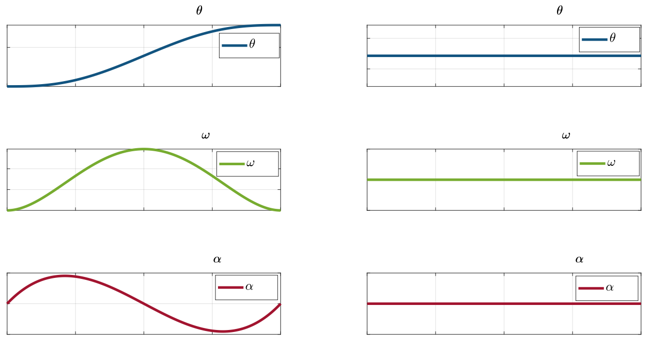


Figure 10. Angular parameters during the movement from the position of the arms defined by point G to the position defined by point H, described by the curve k_4

Examining the movement of the manipulator arm in the position for the maximum extension of the arms in the -x axis and maximum extension in the +x axis (Fig. 11) is further described. It involves the motion from the initial position I to the final position J. With the arm positions shown in Fig. 11, after determining the angular rotations, we define its trajectory k_5 (Fig. 12). We obtain it as the arms move from the initial position defined by point I to the final position defined by point J of the second arm (Table 1). The result is the trajectory profile k_5 in Fig. 12.

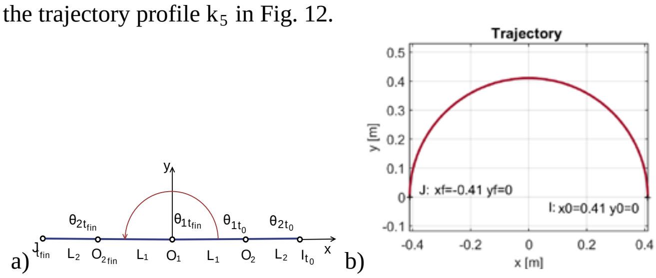


Figure 11. a) The position of the arms at maximum -x and +x. b) trajectory k_5 during the motion from point I to point J.

The progression of angular parameters during the motion of the arms from the initial position I to the final position J is shown in Fig. 12. As expected, the rotation angle of the second arm θ_{2IJ} is zero, and similarly, the angular velocity and angular acceleration of the second arm are zero (Fig. 12).

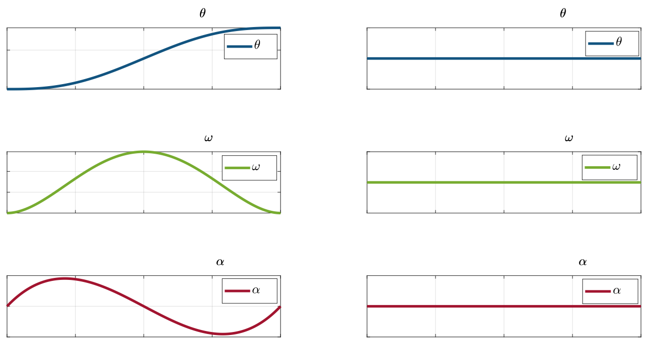


Figure 12. The angular parameters' progression during motion, from the arm position indicated by point I to the arm position specified by point J, described by the curve k_5

The progress of individual trajectories plotted during the movement of the end-effector of the manipulator within the defined workspace is shown in Fig. 13. Angular constraints for the workspace are specified for the angle: $-90° \leq \theta_1 \leq 180°$ and $0° \leq \theta_2 \leq 180°$.

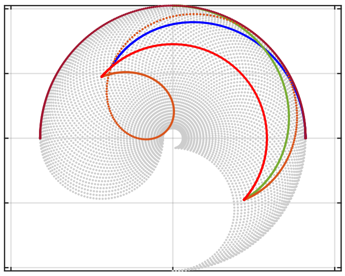


Figure 13. Trajectory and workspace

3. COMPUTER SIMULATION OF FORWARD AND INVERSE DYNAMICS

In the kinematic pairs of the two-link manipulator depicted in Fig. 14, dynamic forces exert influence through the action of driving torques τ_1 a τ_2. These torques are visualized in Fig. 15 and were derived through the solution of the inverse dynamics problem [6-8].

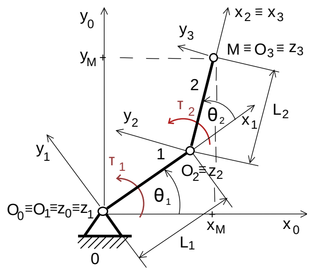


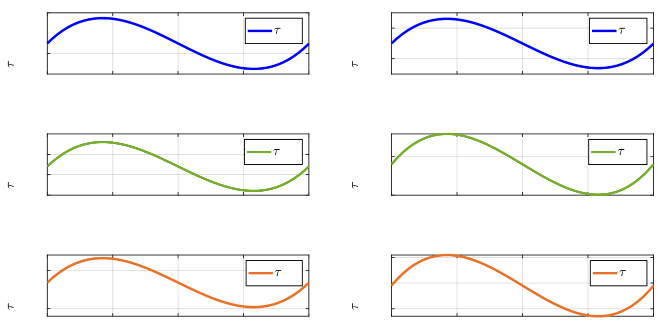
Figure 14. Model of a manipulator with actuating torques at the joints.

We express the equations of motion for the system under consideration (Fig. 14) as follows:

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) = \tau \quad (3)$$

where: τ –vector of actuator torques, M(θ) –inertia matrix, $V(\theta,\dot{\theta})$ –Coriolis centripetal vector and G(θ) –gravity vector.

In the process of designing a robot model, the computation of both kinematic and dynamic variables becomes imperative. The derived values of these parameters serve as a foundation for the detailed design of individual components of the robot. Graphs depicting the computed torque values τ_1 and τ_2 are presented in Fig. 15. The torques were computed through the utilization of the SimMechanics program within the Matlab/Simulink environment [9-11].



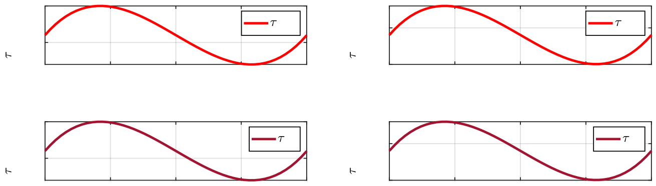


Figure 15. The torques τ_1 and τ_2 during the movement along the trajectories k_1, k_2, k_3, k_4 a k_5

By the presented methodology, the magnitudes of moments in the joints of both arms were determined. Various trajectories of end-point motion of the arms were selected. As expected, maximum values of moment magnitudes were obtained in the segments of motion with the maximum extension of the arms. The mentioned methodology, using Matlab, enables the solution of similar robotics tasks both in practical applications and in teaching the theory of direct and inverse kinematics and direct and inverse dynamics [12-15].

Table 4. Sizes of torques between points A-B (k_1), C-D (k_2), E-F (k_3), G-H (k_4), I-J (k_5)

	A-B (k_1)	**C-D (k_2)**	**E-F (k_3)**	**G-H (k_4)**	**I-J (k_5)**
τ_{1max} (Nm)	0.9601	0.9601	1.1592	1.0002	1.0003
τ_{2max} (Nm)	0.2598	0.2598	0.4183	0.2599	0.26

The possibilities of computer simulation in Matlab were implemented on models of a manipulator with a fixed base. Simulation provides immediate information about the magnitudes of the parameters of the solved model. Computer simulation allows for a quick change of model parameters. Numerical results were processed in the form of clear graphs. The Matlab program is advantageously used for simulating the motion of mechanical systems, industrial robots, and manipulators. The presented methodology provides a suitable tool for addressing issues in teaching and practice.

4. CONCLUSION

The contribution addressed the inverse kinematics problem in the simulation program Matlab, followed by the forward kinematics problem. In the inverse problem, the initial and final angles of the arms were determined for a known position of the end effector. Trajectories of the end effector of a two-link manipulator model were determined, and the profiles of angular variables were established using a fifth-degree polynomial. Forward kinematics leads to the representation of the manipulator's workspace. Solving the inverse dynamics problem determines the torque values in the rotational kinematic pairs, based on which the necessary drive values in individual joints are determined. The Matlab program, along with other simulation programs, serves to expedite, facilitate, and refine the design, innovation, and calculation of various dynamic systems in engineering practice.

References

1. SICILIANO, B., KHATIB, O. 2008. Handbook of Robotics. Heidelberg, Berlin Springer-Verlag.
2. CRAIG, J.J. 2005. Introduction to robotics: mechanics and control. Upper Saddle River: Pearson Prentice Hall.
3. MURRAY, R.M., Li, Z., SASTRY, S.S. 1994. A Mathematical Introduction to Robotic Manipulation. California: University of California: CRC Press.
4. PAUL, R.P. 1981. Robot manipulators, Mathematics, Programming and Control. Cambridge. MA: MIT Press.
5. MIKOVÁ, Ľ., GMITERKO, A., KELEMEN, M., VIRGALA, I., PRADA, E., HRONCOVÁ, D., VARGA, M. 2020. Motion control of nonholonomic robots at low speed. In: International Journal of Advanced Robotic Systems. Wien:Technische Universität Wien, vol. 17, no. 1, pp.1-12.
6. DELYOVÁ, I., FRANKOVSKÝ, P., HRONCOVÁ, D. 2011. Kinematic analysis of movement of a point of a simple mechanism. In: 4th International Conference Modelling of mechanical and mechatronics systems. Košice: KAMaM, Technical University Košice, Herľany, Slovakia.
7. MIKOVÁ, Ľ., GMITERKO, A., FRANKOVSKÝ, P., HRONCOVÁ, D. 2016. Impact of dynamics of the frame on the performance of the positioning servosystem. In: International Journal of Advanced Robotic Systems, vol. 13, no. 5, pp. 1-6.
8. HRONCOVÁ, D., HUŇADY, R., MIKOVÁ, Ľ., SIVÁK, P. 2019. Simulácia v programe Matlab/Simulink. Košice: Technická univerzita v Košiciach, Slovakia.
9. MALÁKOVÁ, S. 2021. Application of glued joints in passenger cars. GRANT Journal. Volume: 9, Issue: 1, p. 106-109. ISSN 1805-062X, 1805-0638 (online), ETTN 072-11-00002-09-4.
10. SÁGA, M., et al. 2020. Case study: Performance analysis and development of robotized screwing application with integrated vision sensing system for automotive industry. International Journal of Advanced Robotic Systems. Vol. 17, No. 3.
11. SAPIETOVÁ, A., et al. 2018. Application of optimization algorithms for robot systems designing. International Journal of Advanced Robotic Systems. Vol.15, No.1, pp. 1-10.
12. TEDESCHI, F., CARBONE, G. 2017. Design of a novel leg-wheel hexapod walking robot. Robotics. 6(4), 4, 2017.
13. TEDESCHI, F., CARBONE, G. 2015. Hexapod Walking Robot Locomotion. Mechanisms and Machine Science. Volume 29, pp. 439-468.
14. CARBONE, G., DI NUOVO A. 2016. A hybrid multi-objective approach for optimal path planning of a hexapod robot a preliminary study. Lecture Notes in Computer Science (including sub-series Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics). Volume 9668, 2016, pp. 131-144.
15. http://www.mathworks.com