

# Construction of the flexspline of harmonic gearing

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**Abstrakt** Flexspline is one of the most important components of harmonic drive. Flexspline is very adversely stressed during operation. In essence, stresses from deformation by the generator, stresses from the transmitted load and local stresses from the bending of the teeth in the tooth gaps act in it. Flexspline is radially flexible, but torsionally very stiff. When the wave generator is inserted into the Flexspline, the gear takes on its elliptical shape. When designing a flexible wheel, it is very important to determine and correctly choose its geometric properties.

**Key words** flexspline, harmonic drive, wave generator, construction.

## 1. INTRODUCTION

A transmission called Harmonic Drive was designed in the USA by Muser to increase torque by reducing the rotational speed of the motor as a power source. Harmonic transmission has a structure with three components: wave generator (wave generator WG), flexible gear wheel (flexspline FS) and rigid gear wheel (circular spline CS) [5]. Harmonic drives have a number of advantages, but also disadvantages, compared to classic gears. The main advantages include: high torque, excellent positioning accuracy and repeatability, compact design, zero backlash, high reduction gear ratio and high torsional stiffness. On the other hand, their disadvantage is high elasticity, non-linear stiffness and damping. The most common way to operate a harmonic gear drive uses a wave generator as the input link and a flywheel as the output link. Due to the fact that one member is flexible in harmonic gears (Fig. 1), these gears have two peculiarities compared to planetary gears. The first peculiarity of harmonic gears consists in the fact that due to the changed shape of the flexible wheel due to the load, or due to the chosen shape of the generator cam, there is a very small relative movement between the teeth located in the gear mesh. The second peculiarity is also conditioned by the shape of the flexible wheel, it consists in the reduction of the pressure angles in the kinematic pair of wave generator - flexible wheel, which is reflected in the reduction of frictional losses of this pair in the comparison of the follower-satellite in the planetary gear. The flywheel is the main part of the harmonic gear, which can generate repetitive vibrations of the wave generator. For this reason, the flexible wheel should have flexibility and good vibration characteristics.

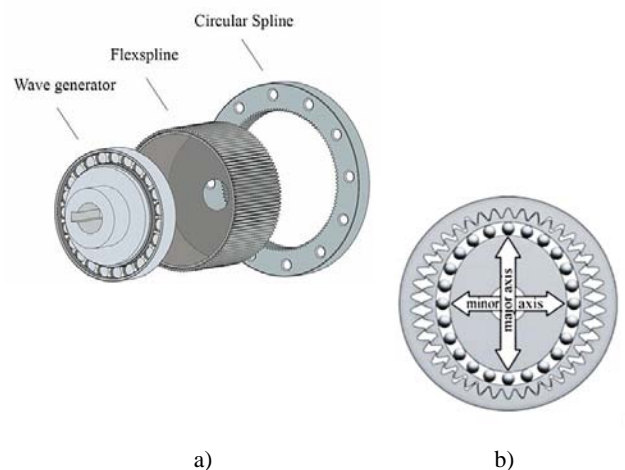


Fig. 1 Design of harmonic drive: a) main elements of harmonic drive; b) interaction of harmonic driving elements

## 2. CONFIGURATION OF WAVE GENERATOR

The wave generator is an elliptical steel disc forming the heart of the gearbox, which has a centric hub and a special thin, elliptically deformable ball bearing. It is also connected to the motor shaft. The Elliptical Wave Generator is a driven element and initiates the Harmonic Drive traction wave process with FS deformation. The latter meshes with an internally toothed circular spline in opposite areas of the major elliptical axis. The rotation of the wave generator causes a displacement of the major elliptical axis along with the captured area of the teeth. The wave generator can have different configurations, each of which deforms the spring wheel in a different way. This means that the deformed shape of the spring wheel is defined by the shape of WG. The following types of wave generator are used in practice:

- cosine - cam WG (Fig. 2 a))
- WG type of two discs (Fig. 2 b))
- two - roller WG (Fig. 2 c))
- four - roller WG (Fig. 2 d))

For the analytical description of the median surface of the flexible gear wheel of individual WG configurations, it is necessary to introduce several assumptions under which the equations described by us will be valid:

- The neutral curve of the flexible wheel has a constant length (does not change during deformation)
- We consider the individual teeth of the flexible wheel to be rigid, deformation occurs only at the root of the tooth
- We consider that the tooth is "cut off" from the surface of the spring wheel in the deformed state of the spring wheel (separation at the root of the tooth).

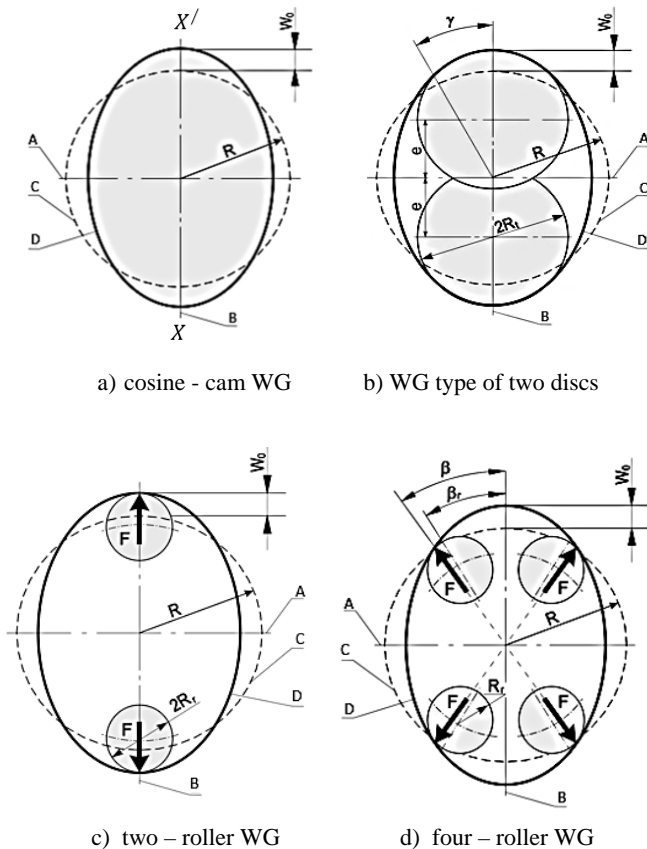


Fig. 2 Schematic representation of typical WG configurations: A – secondary axis of the generator, B – axis of the generator, C – neutral layer of the flexspline before deformation, D - neutral layer of the FS after deformation.

The FS fits closely to the wave generator so that it deforms with the shape of the rotating ellipse, but does not rotate with the wave generator. The FS and the wave generator are located inside a circular groove that wraps around the flex line teeth and the circular groove teeth [3].

Deformation function of the cosine-cam

$$w(\varphi) = w_0 \cos 2\varphi \tag{1}$$

Where  $w_0$  is the maximum pre-deformation amount.  
Deformation function of the roller-cam:

$$w(\varphi) = \frac{w_0}{\sum_{n=2,4,6...} \frac{\cos n\beta}{(n^2 - 1)^2}} \sum_{n=2,4,6...} \frac{\cos n\beta \cos n\varphi}{(n^2 - 1)^2} \tag{2}$$

Where the four-roller WG  $\beta$  value is  $30^\circ$ . Two-roller WG can be regarded as a special case of four-roller WG when  $\beta = 0^\circ$ .

### 3. FLEXSPLINE HARMONIC GEAR

The key element of the harmonic gear used for the transmission of motion is a flexible gear wheel, which is formed by a thin-walled cylinder, while on its outer edge (open) there is an external toothing [10]. In most cases, the closed part of the cylindrical container is reinforced (thicker than the cylindrical surface of the spring wheel), since the output shaft of the gearbox transmitting high torques is directly connected to it.

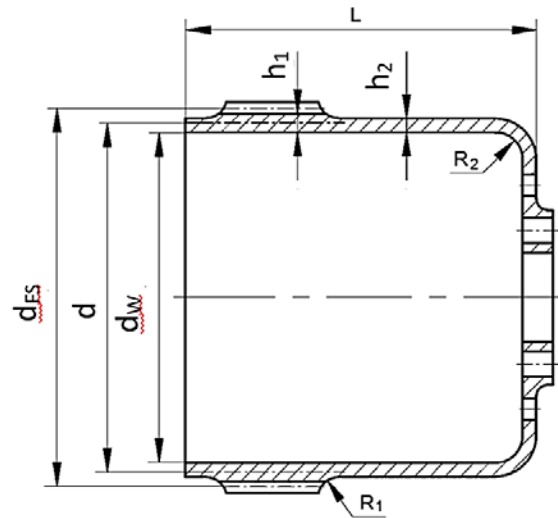


Fig. 3 The flexspline geometric model:  $d_{FSW}$  – spacing diameter,  $d$  - diameter of the neutral layer circle,  $d_w$  - inside diameter,  $h_1$  - body thickness without toothed ring,  $h_2$  - sleeve body thickness,  $L$  - flexspline length.

The shape of the spring wheel (Fig. 3) depends on the type of wave generator, which is inserted into the open part of the spring wheel during assembly [8]. Holes are indicated through the closed part of the cylindrical container. With the help of screws guided through these holes, the output member of the gear is subsequently mounted, to which the loads can be connected in the necessary manner.

The flexible wheel undergoes constant deformation from the wave generator while the gearbox is running, and at the same time there is multiple contact in the teeth between the flexible wheel and the rigid wheel. As a result, the correct choice of the material of the flexible wheel is important.

#### 3.1 Material of flexspline

When designing a flexible wheel, it is very important to determine and correctly choose its geometric properties. Adequate selection of the dimensions of the flexible wheel should ensure the minimization of stresses in dangerous cross-sections and a more constant distribution of the stress of the flexible wheel, which must be

flexible in the radial direction but rigid in the torsional direction. These phenomena cannot be avoided when the spring wheel is made of conventional isotropic materials such as steel [6].

Therefore, the appropriate application of materials or technological modifications is very important. Using composites on the FS enables weight reduction of the FS and increases significantly its radial susceptibility and damping of vibration [5]. The production of FS made entirely from composites is, however, constrained by technological difficulties connected with creating their toothed ring gear. This problem may be solved by the application of so-called complex steel-composite hybrid FS (Fig. 4) [3].

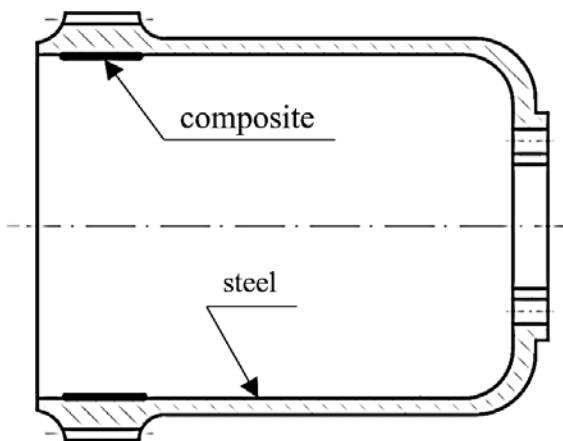


Fig. 4 The construction of the so-called "pot" shaped flexible wheel made of steel - composite material.

#### 4. TYPES OF FLEXSPLINE

The geometric parameters of the constructions that we most often encounter are shown in fig. 3. Classic flexible wheels can have external (Fig. 5a) or internal (Fig. 5b)) flanges.

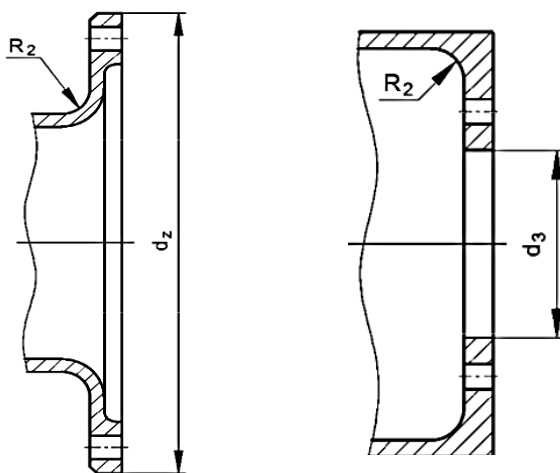
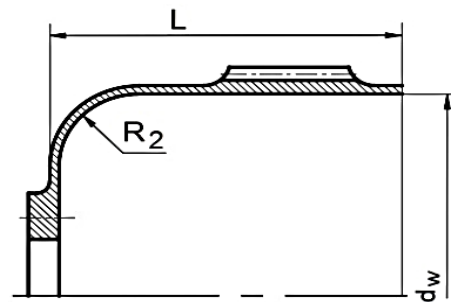
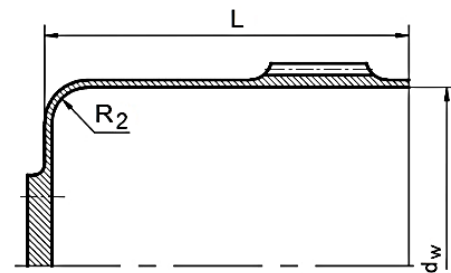


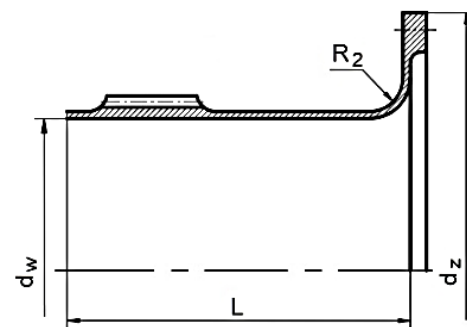
Fig. 5 Construction of flexspline of harmonic gear wheel:  $d_z$  – diameter of the outer flange,  $d_3$  – diameter of inner flange,  $R_2$  – radius of rounding a) outer flange, b) inner flange.



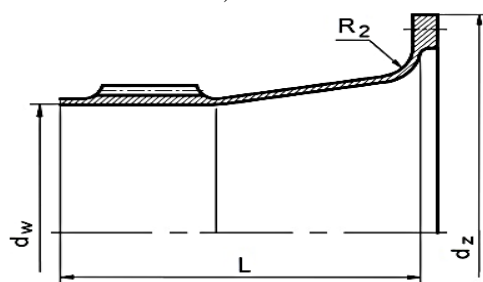
a)



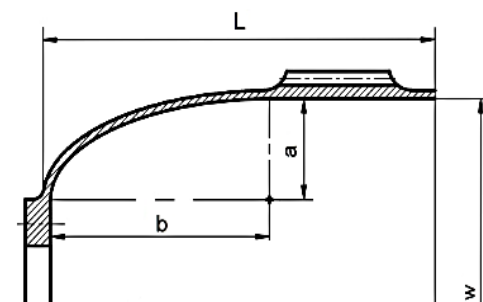
b)



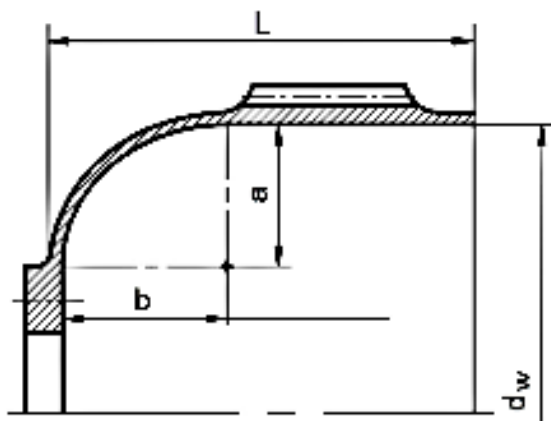
c)



d)



e)



f)

Fig. 6 Basic types of flexspline of harmonic transmission

The existence of a FS in a gear transmission, deforming during work, needs a special approach to the investigation of engagement in this gear. Above all, it is necessary to express the influence of the deformation of the FS the shape of the teeth distributed on it.

Flexspline of harmonic gear has elastic properties and depending on the wave generator can take on specific shapes. In fig. 6 shows the most frequently used forms of the flexible harmonic gear wheel. Many of the flexible wheel shapes that have been investigated have proven to be completely useless due to their low resistance. The goal of various analyzes of harmonic gear flywheel constructions is to obtain a shape of the flywheel that would improve its service life and reliability.

Based on the experimental results, it was found that with the possible values of deformations of the FS and real ratios between the diameter of the central surface and the thickness of the wall of the elastic wheel, there is no significant change in the shape of the profiles and the position of the teeth [7, 8]. We call the middle surface the geometric location of the points dividing the wall thickness of the flexible wheel in half. In fig. 7. the shape of the teeth is indicated in the section of the largest deformation of the gear ring - at point X - X' in fig. 2 a).

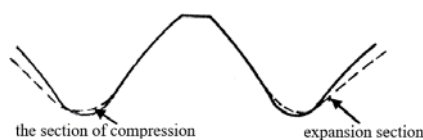


Fig. 7 The shape of the teeth in the part of the greatest deformation

As part of the design, the teeth of the Flexspline are preloaded against those of the circular spline so that any wear in the teeth is compensated for and the drives operate with virtually zero backlash [11]. Friction losses and wear are, in any event, negligible because of the way the teeth come into contact in almost a pure radial motion with almost no sliding velocity.

## 5. REQUIREMENTS PLACED ON TOOTH PROFILES

Since HD was invented by Musser in 1955, there have been extensive researches on tooth profile design and kinematic analysis. Looking at the history of HD, the tooth profile has gradually evolved from straight line [4], involution [5] and S-shaped [6] to double circular arch [7] and cycloid [8]. Regarding HD kinematic analysis, the inventor Musser only schematically showed the working principle of HD in his patent [3] and did not solve the kinematic problem. Scientist Ivanov modeled the relative motion between circular spline and FS as pure rolling friction of circular and non-circular curves on the neutral layer and defined the circumferential ratio of the two curves as the gear ratio.

The researches provide an effective guideline to reveal the movement mapping between the intake end and the exit end of the FS cup. It should be noted that the transmission ratio defined by the two kinematic models is actually the average speed ratio of the input wave generator and the output end of the FS [13, 14]. Considering that the WG forces the FS to produce elastic deformation and the FS tooth undergoes a small oscillation, the defined gear ratio obviously cannot accurately describe the engagement between the CS and FS teeth.

The pressure angle of the gear teeth transforms the output torque's tangential force into a radial force acting on the wave-generator bearing. The teeth of the FS and circular spline engage near the ellipse's major axis and disengage at the ellipse's minor axis. It is known the FS has two less teeth than the circular spline, so every time the wave generator rotates one revolution, the flexspline and circular spline shift by two teeth.

The main purpose of adjusting the tooth profile is to reduce the impact load between the teeth, the number of engaged teeth is large at the same time, about 20~30% of the FS teeth are engaged, and the engagement state of each tooth is different (Fig. 8). Finally, for the same tooth, the engagement condition is not the same in different axial positions.

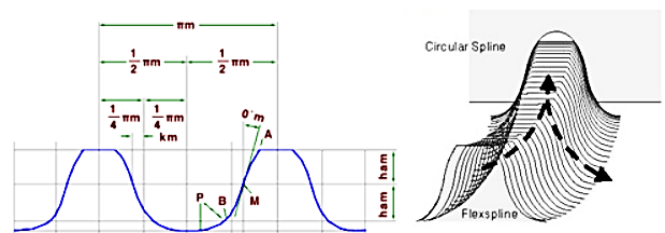


Fig. 8 This is a progression of flex-spline tooth engagement with circular-spline teeth.

Once that the centrodes are determined and that the shape of one tooth is fixed, the shape of the other tooth can be found according to the conjugate action theory. There also many constraint such as interference, strength of materials, surface maximum curvature, ease of manufacturing. The peculiarity of harmonic drive teeth is that they undergo a roto-translational motion, not simply rotational as in traditional gearing. The equivalent curve of FS can be taken as the neutral axis of the rim width [14, 13]. The line passing through the middle of the tooth height can be called "reference circle" or "pitch curve". This line undergoes a slight stretch that cannot be neglected in the synthesis of conjugate action between meshing teeth.

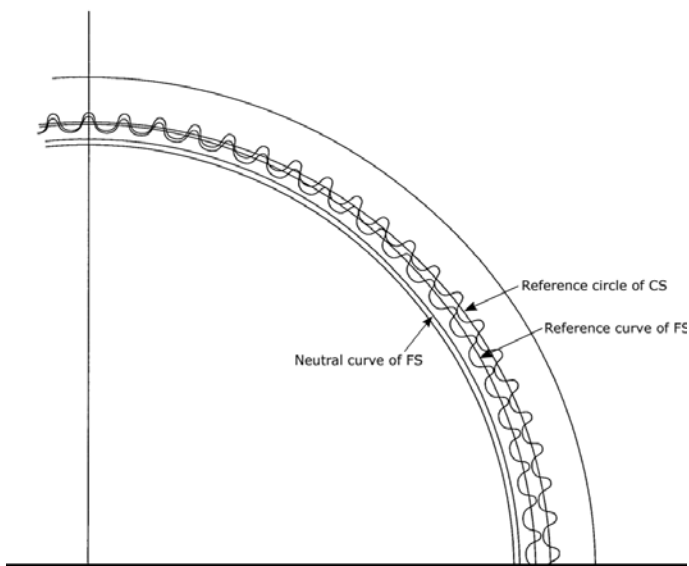


Fig. 9: Drawing of neutral line and of the reference curve or pitch circle.

After determining the shape of the deformed tooth, the task is to design a suitable shape of the counter profile so that there is no interference when the flexible wheel engages with the rigid wheel of the harmonic gear. The flanks of the teeth of the rigid gear must be the envelope curves of the flanks of the teeth of the flexible gear.

## 6. CONCLUSION

The key element of harmonic transmission serving to transmit motion is a flexible gear wheel. The shape of the spring wheel depends on the type of wave generator that is inserted into the open part of the spring wheel during assembly. The spring wheel is under very unfavorable stress during operation. It is affected by stresses from deformation by the generator, stresses from the transmitted load and local stresses from the bending of the tooth in the tooth gaps. The amount of deformation of the flexible wheel has a significant influence on the character and quality of the shot. The analysis of the shape of the flexible wheel makes it possible to assess the possibility of reducing the dimensions of harmonic drives without limiting their strength. The shape of the active sides of the teeth of the fixed wheel should be such that in the place of the greatest load of the flexible wheel, after deformations, there is no increase in tension due to incorrect engagement. Future work can focus on finding out the shape of the deformed tooth of the spring wheel and subsequently design a counter profile so that there is no interference when the spring wheel engages with the rigid harmonic gear wheel.

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